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COMPUTATION OF UNSTEADY FLOW PAST A CYLINDER INSTANTANEOUSLY SET IN MOTION

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§1. The first results on the solution of unsteady flow past a body of finite dimensions instantaneously set in motion were obtained within the framework of the boundary-layer theory.

For the initial flow stage the first two terms of the power series expansion of the solution in the powers of t (t is time) were obtained by Blasius in [1], the obtained solution being valid as $\mathrm{Re} \to \infty$.

The solution found by Blasius was improved in [2]. Subsequently, an attempt was made to extend the Blasius solution to the case of low Reynolds numbers [3, 4].

The use of numerical methods to solve nonstationary Navier–Stokes equations [5-10] turns out to be a more promising approach to the problem under investigation. In [10] a survey of the literature on this subject is given. In the case of suddenly arising motion of a cylinder one of the difficulties lies in the formulation of the initial conditions.

It follows from the theory of the boundary layer [11] that the vorticity of the fluid flow is infinitely large at the initial time instant and is then concentrated in an infinitely thin region around the cylinder surface. Therefore, a straightforward application of finite-difference approximations to the original equations does not produce a correct pattern of the initial flow past the cylinder [7]. Moreover, it was shown in [12] that to obtain in this case a satisfactory approximate solution very small steps in time must be taken.

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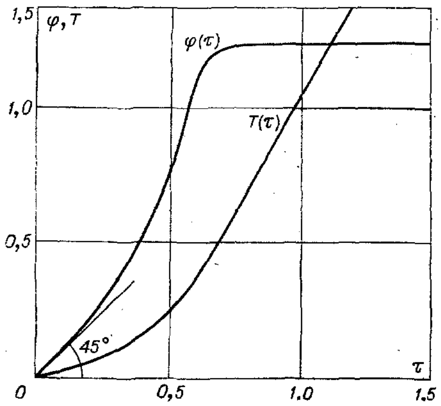

Fig. 1

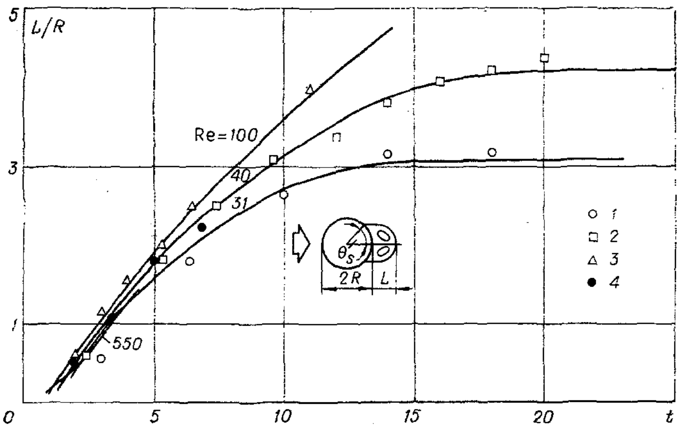

Fig. 2

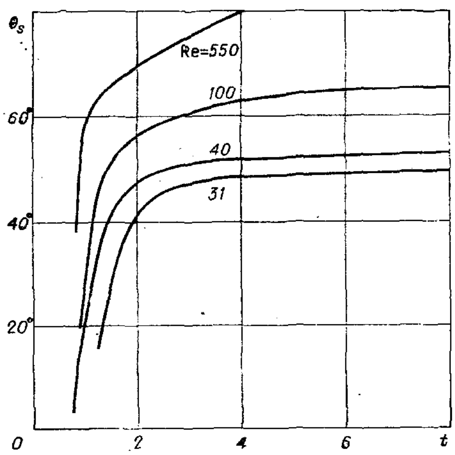

Fig. 3

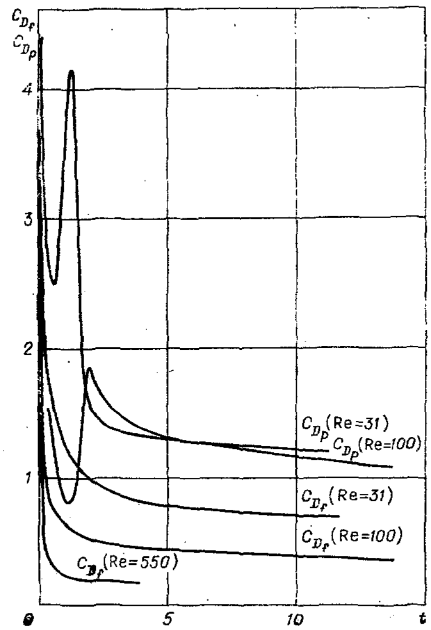

Fig. 4

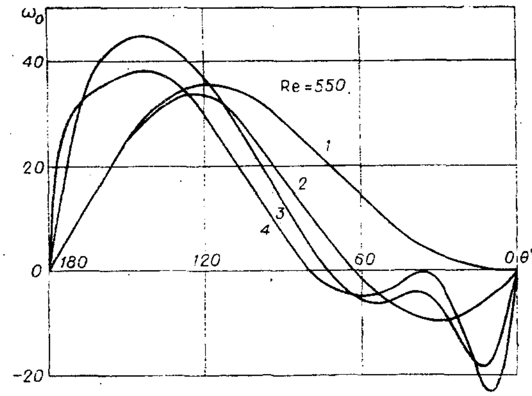

Fig. 5

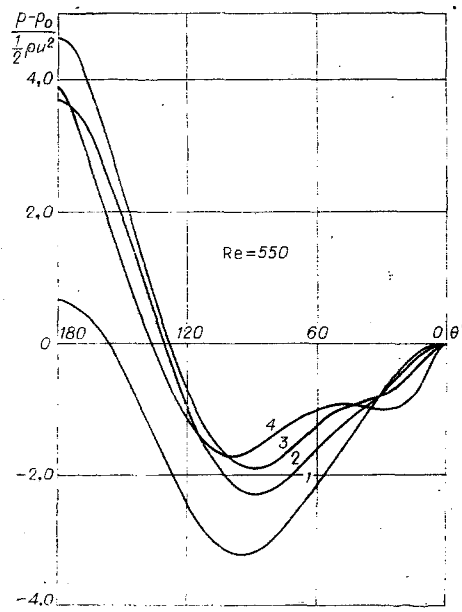

Fig. 6

The first correct formulation of the initial stage of a flow past a cylinder which is set instantaneously in motion appears to have been proposed in [9, 10]; however, the actual numerical method used there does not enable one either to estimate the error in the obtained results or to investigate the flow for longer durations.

Using the numerical method described here one is able to compute in a unified manner the flow from the instant of impulsive start up to and including the coming to rest.

§2. Let a circular cylinder be set instantaneously in motion with velocity u perpendicular to the cylinder axis. It is assumed that the flow past the cylinder is symmetrical. The original Navier – Stokes equations which describe the flow of a viscous incompressible fluid past the cylinder in the polar coordinate system (r, θ) in its dimensionless form are given by

$$\begin{gathered}\frac{\partial\omega}{\partial t}+\frac{1}{r}\frac{\partial\psi}{\partial\theta}\frac{\partial\omega}{\partial r}-\frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial\omega}{\partial\theta}=\frac{2}{\mathrm{Re}}\left(\frac{\partial^2\omega}{\partial r^2}+\frac{1}{r}\frac{\partial\omega}{\partial r}+\frac{1}{r^2}\frac{\partial^2\omega}{\partial\theta^2}\right),\\ \omega=\frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right)+\frac{1}{r}\frac{\partial^2\psi}{\partial\theta^2}\right),\end{gathered} \tag{2.1}$$

where ω is the vortex; ψ is the flow function; and $\mathrm{Re} = u2R/\nu$ is the Reynolds number (R is the cylinder radius; ν is the kinematic viscosity).

The boundary conditions are given by

$$\begin{aligned}&\psi=0,\ \partial\psi/\partial r=0 \quad \text{for} \quad r=1, \quad t\geqslant 0;\\ &\omega\to 0, \quad \psi\to r\sin\theta \quad \text{for} \quad r\to\infty, \quad t>0;\\ &\omega\to 0, \quad \psi=0 \quad \text{for} \quad r\to\infty, \quad t=0;\\ &\psi=0, \quad \omega=0 \quad \text{for} \quad \theta=0,\ \pi,\ t\geqslant 0.\end{aligned} \tag{2.2}$$

The starting system of equations (2.1) is now transformed by employing the following considerations in physics. It is known [11] that in the case of instantaneously arising motion of a body the boundary layer forming on the body is of a thickness which varies in time as $2\sqrt{2t/\mathrm{Re}}$ (the relation is valid for small t). Moreover, with the boundary layer (or its vorticity) developing when the convection is the decisive transfer mechanism of vorticity and the layer is of suitable thickness, the thickness of the boundary layer changes according to the law e^{ct} ($c = 1/2u$) [13].

Taking the above into account, the following transformation of the independent variables is introduced:

$$\begin{gathered}r=e^{\alpha\pi k(\tau)\xi},\quad \theta=\pi\eta,\\ t=T(\tau),\end{gathered} \tag{2.3}$$

where $k(\tau) = 2\sqrt{2/\mathrm{Re}}\,\varphi(\tau)$; the shapes of the functions $\varphi(\tau)$ and $T(\tau)$ are shown in Fig. 1. To be able to eliminate the initial singularity of the eddy function one introduces the normalized eddy W and the normalized flow function by means of the formulas

$$\psi = k(\tau)\Psi(\eta, \xi, \tau), \ \omega = W(\eta, \xi, \tau)/k(\tau). \tag{2.4}$$

By using (2.3) and (2.4), Eqs. (2.1) become

$$P(\tau)\frac{\partial W}{\partial \tau} + \frac{k^2(\tau)}{2\alpha E^2}\frac{D(\Psi, W)}{D(\eta, \xi)} = \frac{2}{\mathrm{Re}}\left(\frac{1}{2\alpha^2 E^2}\frac{\partial^2 W}{\partial \xi^2} + Q(\tau)\xi\frac{\partial W}{\partial \xi} + Q(\tau)W + \frac{k^2(\tau)}{2E^2}\frac{\partial^2 W}{\partial \eta^2}\right),$$
$$E^2 W = \frac{1}{\alpha^2}\frac{\partial^2 \Psi}{\partial \xi^2} + k^2(\tau)\frac{\partial^2 \Psi}{\partial \eta^2}, \tag{2.5}$$

where

$$P(\tau) = \frac{k^2(\tau)}{2\beta}; \quad \beta = \frac{dT}{d\tau}; \quad Q(\tau) = \frac{\mathrm{Re}}{4}\frac{k(\tau)k'_\tau(\tau)}{\beta};$$
$$D(\Psi, W)/D(\eta, \xi) = (\partial\Psi/\partial\eta)\,\partial W/\partial\xi - (\partial\Psi/\partial\xi)\,\partial W/\partial\eta; \quad E = \pi e^{\alpha\pi k(\tau)\xi},$$

α being the constant parameter. The coefficients P and Q are continuous functions of τ. The transformation $t = T(\tau)$ is chosen in such a way that the required condensation of the time scale is ensured at the neighborhood of $t = 0$.

If one takes into account (2.3) and (2.4), the boundary conditions (2.2) take the form

$$\Psi = 0, \ \partial\Psi/\partial\xi = 0 \quad \text{for} \quad \xi = 0, \ \tau \geqslant 0;$$
$$W = 0, \ \partial\Psi/\partial\xi = \alpha\pi e^{\alpha\pi k(\tau)\xi_{\max}} \sin\pi\eta \quad \text{for} \quad \xi = \xi_{\max}, \tau > 0; \tag{2.6}$$
$$W = 0, \ \Psi = 0 \quad \text{for} \quad \eta = 0, 1, \ \tau \geqslant 0,$$

where $\xi_{\max}$ is a suitably large value of ξ.

Equations (2.5) together with the boundary conditions (2.6) constitute a closed system for the determination of W, Ψ for $\tau > 0$.

§3. It can be shown that for $\tau = 0$ the system (2.5) assumes the self consistent form

$$(1/2\alpha^2\pi^2)\partial^2 W/\partial\xi^2 + \xi\partial W/\partial\xi + W = 0,$$
$$W = (1/\alpha^2\pi^2)\partial^2\Psi/\partial\xi^2 \tag{3.1}$$

with the boundary conditions

$$\Psi = 0, \ \partial\Psi/\partial\xi = 0 \quad \text{for} \quad \xi = 0;$$
$$W = 0, \ \partial\Psi/\partial\xi = \alpha\pi\sin\pi\eta \quad \text{for} \quad \xi = \xi_{\max}. \tag{3.2}$$

The solution of the first equation of (3.1) which satisfies the conditions (3.2) can be represented in the form

$$W = C(\eta)\,e^{-(\alpha\pi\xi)^2}.$$

To determine $C(\eta)$ the second equation of (3.1) is integrated with respect to ξ yielding

$$\left(\frac{\partial\Psi}{\partial\xi}\right)_{\xi=\xi_{\max}} - \left(\frac{\partial\Psi}{\partial\xi}\right)_{\xi=0} = (\alpha\pi)^2 C(\eta)\int\limits_0^{\xi_{\max}} e^{-(\alpha\pi\xi)^2}d\xi.$$

By using (3.2), one finds

$$C(\eta) = \frac{2}{\alpha\pi^{3/2}}\left(\frac{\partial\Psi}{\partial\xi}\right)_{\xi=\xi_{\max}} = \frac{2}{\sqrt{\pi}}\sin\pi\eta.$$

Thus

$$W(\eta, \xi, 0) = \frac{2}{\sqrt{\pi}}e^{-(\alpha\pi\xi)^2}\sin\pi\eta \tag{3.3}$$

is the required initial condition for the eddy.

The corresponding initial condition for the flow function is easily found from the solution of the second equation of (3.1) by using (3.3).

§4. Let an orthogonal grid with subdivision steps $h_1 = \Delta\eta$, $h_2 = \Delta\xi$, $\Delta\tau$ be defined in the integration region. The finite-difference scheme for the employed numerical method is given by

$$P_m \frac{3W_{ij}^{n-\frac{1}{2}m} - 4W_{ij}^{m-1} + W_{ij}^{m-2}}{2\Delta\tau} + \frac{1}{2}\frac{k_m^2}{\alpha E_{mj}^2}\left(\frac{W_{ij+1}^{rm} - W_{ij}^{rm}}{2h_2} + \right.$$

$$\left. + \frac{W_{ij}^{sm} - W_{ij-1}^{sm}}{2h_2}\right)\left(\frac{\Psi_{i+1j}^{n-1m} - \Psi_{i-1j}^{nm}}{2h_1}\right) - \frac{1}{2}\frac{k_m^2}{\alpha E_{mj}^2}\left(\frac{W_{i+1j}^{pm} - W_{ij}^{pm}}{2h_1} + \frac{W_{ij}^{qm} - W_{i-1j}^{qm}}{2h_1}\right)\left(\frac{\Psi_{ij+1}^{n-1m} - \Psi_{ij-1}^{n-1m}}{2h_2}\right) =$$

$$= \frac{2}{\mathrm{Re}}\left(\frac{1}{2\alpha^2 E_{mj}^2}\frac{W_{ij+1}^{n-\frac{1}{2}m} - 2W_{ij}^{n-\frac{1}{2}m} + W_{ij-1}^{n-\frac{1}{2}m}}{h_2^2} + Q_m \xi_j \frac{W_{ij+1}^{n-\frac{1}{2}m} - W_{ij-1}^{n-\frac{1}{2}m}}{2h_2} + \right.$$

$$\left. + Q_m W_{ij}^{n-1m} - \frac{W_{ij}^{n-\frac{1}{2}m} - W_{ij}^{n-1m}}{\varkappa_1} + \frac{k_m^2}{2E_{mj}^2}\frac{W_{i+1j}^{n-1m} - 2W_{ij}^{n-\frac{1}{2}m} + W_{i-1j}^{nm}}{h_1^2}\right); \tag{4.1}$$

$$\Psi_{ij+1}^{n-\frac{1}{2}m} - 2\left(1 + \alpha^2 k_m^2 \frac{h_2^2}{h_1^2}\right)\Psi_{ij}^{n-\frac{1}{2}m} + \Psi_{ij-1}^{n-\frac{1}{2}m} = \alpha^2 h_2^2 E_{mj}^2 W_{ij}^{nm} -$$

$$- \alpha^2 k_m^2 \frac{h_2^2}{h_1^2}\left(\Psi_{i+1j}^{n-1m} + \Psi_{i-1j}^{nm}\right); \quad W_{ij}^{nm} = \varkappa_2 W_{ij}^{n-\frac{1}{2}m} + (1-\varkappa_2) W_{ij}^{n-1m};$$

$$\Psi_{ij}^{nm} = \varkappa_3 \Psi_{ij}^{n-\frac{1}{2}m} + (1-\varkappa_3)\Psi_{ij}^{n-1m},$$

where

$$W_{ij}^{nm} = W(ih_1, jh_2, m\Delta\tau); \quad \Psi_{ij}^{nm} = \Psi(ih_1, jh_2, m\Delta\tau);$$
$$P_m = P(m\Delta\tau); \quad Q_m = Q(m\Delta\tau); \quad E_{mj} = \pi e^{\alpha\pi k_m \xi_j};$$
$$\xi_j = jh_2; \quad k_m = k(m\Delta\tau),$$

(n is the iteration superscript; $\varkappa_1$ is the regularization parameter; and $\varkappa_2$, $\varkappa_3$ are relaxation parameters). The superscripts r, s, p. q are chosen depending on the sign of

$$u_{ij} = -\frac{\Psi_{ij+1}^{n-1m} - \Psi_{ij-1}^{n-1m}}{2h_2}, \quad v_{ij} = \frac{\Psi_{i+1j}^{n-1m} - \Psi_{i-1j}^{nm}}{2h_1}$$

in the following manner:

$$s = n - 1/2, r = n - 1, \ q = n - 1/2, p = n - 1 \quad \text{for} \quad u_{ij} \geqslant 0, \ v_{ij} \geqslant 0;$$
$$s = n - 1, \ r = n - 1/2, q = n - 1, \ p = n - 1/2 \quad \text{for} \quad u_{ij} < 0, \ v_{ij} < 0;$$
$$s = n - 1/2, r = n - 1, \ q = n - 1, \ p = n - 1/2 \quad \text{for} \quad u_{ij} < 0, \ v_{ij} \geqslant 0;$$
$$s = n - 1, \ r = n - 1/2, q = n - 1/2, p = n - 1 \quad \text{for} \quad u_{ij} \geqslant 0, \ v_{ij} < 0.$$

The difference scheme under consideration possesses an error of the second order in the space coordinates and in time.

The system of equations (4.1) can also be represented in the form

$$A_{ij}^{n-1m} W_{ij+1}^{n-\frac{1}{2}m} + B_{ij}^{n-1m} W_{ij}^{n-\frac{1}{2}m} + C_{ij}^{n-1m} W_{ij-1}^{n-\frac{1}{2}m} = D_{ij}^{n-1m};$$
$$\Psi_{ij+1}^{n-\frac{1}{2}m} + R_m \Psi_{ij}^{n-\frac{1}{2}m} + \Psi_{ij-1}^{n-\frac{1}{2}m} = S_{ij}^{n-1m}; \tag{4.2}$$
$$W_{ij}^{nm} = \varkappa_2 W_{ij}^{n-\frac{1}{2}m} + (1-\varkappa_2) W_{ij}^{n-1m};$$
$$\Psi_{ij}^{nm} = \varkappa_3 \Psi_{ij}^{n-\frac{1}{2}m} + (1-\varkappa_3)\Psi_{ij}^{n-1m},$$

where the coefficients A_{ij}^{n-1m}, B_{ij}^{n-1m}, C_{ij}^{n-1m}, D_{ij}^{n-1m}, S_{ij}^{n-1m} are functions of W_{ij}^{m-1}, W_{ij}^{m-2}, W_{ij}^{n-1m}, Ψ_{ij}^{n-1m}.

Thus, the difference scheme (4.2) is a modification of the method of the upper relaxation block [14]. The first two equations of (4.2) are solved by using linear factorization [15].

It can be shown that the difference scheme (4.1) is stable and monotonic. For τ = 0 the scheme (4.1) becomes

$$\frac{W_{ij+1}^{n-\frac{1}{2}} - 2W_{ij}^{n-\frac{1}{2}} + W_{ij-1}^{n-\frac{1}{2}}}{h_2^2} + \xi_j \frac{W_{ij+1}^{n-\frac{1}{2}} - W_{ij-1}^{n-\frac{1}{2}}}{2h_2} + W_{ij}^{n-1} = \frac{W_{ij}^{n-\frac{1}{2}} - W_{ij}^{n-1}}{\varkappa_1}$$

or

$$aW_{ij+1}^{n-\frac{1}{2}} - bW_{ij}^{n-\frac{1}{2}} + cW_{ij-1}^{n-\frac{1}{2}} = -d, \tag{4.3}$$

where

$$a = \frac{\varkappa_1}{h_2^2} + \xi_j \frac{\varkappa_1}{2h_2}; \quad c = \frac{\varkappa_1}{h_2^2} - \xi_j \frac{\varkappa_1}{2h_2};$$

$$b = \frac{2\varkappa_1}{h_2^2} + 1; \quad d = (\varkappa_1 + 1) W_{ij}^{n-1}.$$

The condition that the scheme be monotonic imposes the following constraints:

$$a > 0, \ b > 0, \ c > 0, \ b \geqslant a + c, \tag{4.4}$$

and hence it follows that

$$c = \frac{\varkappa_1}{h_2^2} - \xi_j \frac{\varkappa_1}{2h_2} > 0, \quad h_2 < \frac{2}{\xi_{max}}. \tag{4.5}$$

If the conditions (4.4) and (4.5) are satisfied, it follows by virtue of the maximum theorem [15] that the implicit difference scheme (4.3) is stable.

One employs the Woods condition [16] for the boundary condition for the eddy on the body,

$$W_{i0}^{nm} = \frac{3\Psi_{i1}^{n-1m}}{(\alpha\pi h_2)^2} - \frac{W_{i1}^{n-1m}}{2} e^{2\alpha\pi k_m h_2} + O(h_2^2),$$

and at every iteration the correcting of the boundary condition for the vorticity on the cylinder surface is carried out by means of the formula (4.6). Numerical scanning of the computation region at each time step m takes place until the convergence condition

$$\max_{ij} \left\{ \left| \frac{\Delta\Psi_{ij}^{nm}}{\Psi_{ij}^{nm}} \right|, \left| \frac{\Delta W_{ij}^{nm}}{W_{ij}^{nm}} \right| \right\} \leqslant 10^{-4}$$

has been satisfied, where

$$\Delta\Psi_{ij}^{nm} = \Psi_{ij}^{nm} - \Psi_{ij}^{n-1m}; \quad \Delta W_{ij}^{nm} = W_{ij}^{nm} - W_{ij}^{n-1m}.$$

§5. To verify the accuracy of the numerical method used here a comparison was made between the calculation results for $\tau = 0$ and the exact solution. The highest percentage difference between the numerical solution and the exact one for a network with steps $h_1 = 0.033$, $h_2 = 0.2$ and $\xi_{max} = 6$ does not exceed 0.01%. To be able to evaluate the effect of the location of the outer boundary on the numerical solution ξ_{max} was varied.

In the present work the computations of an unsteady flow past a cylinder which is set instantaneously in motion with constant velocity were carried out by using the described numerical method. Some computation results are shown in Figs. 2-6.

In our calculations the following alternatives were used: Re = 31; 40; 100; 550. The results of the numerical calculations were compared with the experimental results obtained in [17].

In Fig. 2 the comparison results are shown, the following notation being adopted for the experimental points of [17]:

1 — Re = 31; *2* — Re = 40; *3* — Re = 100; *4* — Re = 550.

In Fig. 3 the separation angle θ_s is shown versus time with instantaneous acceleration of the cylinder for different Reynolds numbers.

In Fig. 4 the coefficients of friction C_{D_f} and the coefficients of pressure C_{D_p} versus time are shown for different Re.

In Fig. 5 the eddy distribution is shown on the cylinder surface at different time instants for Re = 550, the curves 1-4 corresponding to t = 0.75; 1.25; 2.61; 4.01.

In Fig. 6 the distribution is shown of the dimensionless pressure $2(p-p_0)/\rho u^2$ (p_0 is the pressure at the rear critical point) on the cylinder surface at different time instants for Re = 550. The curves 1-4 in Fig. 6 correspond to t = 1.25; 1.75; 2.25; 4.01.

The following expressions were used for the quantities shown in Figs. 4 and 6:

$$\frac{2(p-p_0)}{\rho u^2} = -\frac{4}{\alpha k^2 \mathrm{Re}} \int_0^{\eta} \left(\frac{\partial W}{\partial \xi}\right)_{\xi=0} d\eta;$$

$$C_{D_f} = \frac{4\pi}{k\,\mathrm{Re}} \int_0^1 W(0, \eta, \tau) \sin \pi\eta d\eta;$$

$$C_{D_p} = -\pi \int_0^1 p(0, \eta) \cos \pi\eta d\eta.$$

More detailed graphical and tabular results illustrating the data obtained from the numerical computations of the unsteady flow past a cylinder set instantaneously in motion can be found in [18].

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